Set-C

Group-B

12

Solution

(i) Given, $z = \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$.

$= \frac{(1+3i)(1+2i)}{1-(2i)^2} = \frac{1+2i+3i+6i^2}{1-4i^2}$.

$= \frac{1+5i-6}{5} = \frac{5i-5}{5} = i-1$.

Thus $a+ib = -1+i$.

(ii)

(iii) Compare $-1+i$ with $x+iy$:

$\therefore x = -1$ and $y = 1$

$r = \sqrt{x^2+y^2} = \sqrt{(-1)^2+(1)^2} = \sqrt{2}$.

(iii) $\tan\theta = \frac{y}{x} = \frac{1}{-1} = -1 = \tan(180° - 45°) = \tan\frac{3\pi}{4}$.

$\theta = 3\pi/4$.

Polar form: $z = r[\cos\theta + i\sin\theta]$

$= \sqrt{2}\,[\cos 3\pi/4 + i\sin 3\pi/4]$

To find square root:- $z^{1/2} = \{\sqrt{2}[\cos 3\pi/4 + i\sin 3\pi/4]\}^{1/2}$.

$= \sqrt{2}^{1/2}[\cos 3\pi/8 + i\sin 3\pi/8]$

$= 2^{1/4}$

13 a.

Ans:- The principal of mathematical Induction states that if $P(n)$ be the statement and if
(i) $P(1)$ is true.
(ii) $P(k+1)$ is true whenever $P(k)$ is true.
then $P(n)$ is true for all $n \in N$.

Solution.

Let $P(n)$ be the given statement than,
$P(n) = 3^{2n} - 1$.
when $n=1$, $P(1) = 3^{2\times 1} - 1 = 8$. is divisible by 8.
$\therefore P(1)$ is true.
Let us suppose that $P(k)$ is true for $k \in N$.
$P(k) = 3^{2k} - 1$. is divisible by 8 ----- (i).
Now, we shall show that $P(k+1)$ is true whenever $P(k)$ is true. Now show, $3^{2(k+1)} - 1$ is divisible by 8.
we have,

$$3^{2(k+1)} - 1 = 3^{2k+2} - 1 = 3^{2k} \cdot 9 - 9 + 9 - 1$$
$$= (3^{2k} - 1)\,9 + 8.$$
$$= 9\,(3^{2k} - 1) + 8$$

Which is divisible by 8 as in the first term is divisible by 8 by (i).
This relation show that $P(k+1)$ is true whenever $P(k)$ is true. Hence by the the principle of mathematical Induction $P(n)$ is true for all $n \in N$.

(b)

Solution.

Let α and 2α be the root of quadratic equation $ax^2 + bx + c = 0$.

(d) 14

$\alpha + 2\alpha = \frac{-b}{a}$ $\qquad$ $\alpha . 2\alpha = \frac{c}{a}$.

$3\alpha = \frac{-b}{a}$ $\qquad$ $2\alpha^2 = \frac{c}{a}$.

$\alpha = \frac{-b}{3a}$ ---(i) $\qquad$ $2\left(\frac{-b}{3a}\right)^2 = \frac{c}{a}$. [from eqn (i)

or, $\boxed{\cancel{\frac{2b^2}{9a}} = \cancel{\frac{c}{a}}}$

or, $\frac{2b^2}{9a^{\cancel{2}}} = \frac{c}{\cancel{a}}$

$\therefore 2b^2 = 9ac$.

proved

14 (b).

solution.

Equation of the plane through $(3,2,1)$ is.

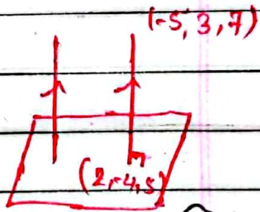
$A(x-3) + B(y-2) + C(z-1) = 0$ ------ (i).

The direction Cosine of the line joining point $(-5,3,7)$ and $(2,-4,5)$ are propotional to.

$7, -7, -2$. $\quad [\because x_2-x_1, y_2-y_1, z_2-z_1]$

The plane (i) is Perpendicular to this ~~plane~~ line. If any normal to the plane is parallel to this line. $\because$ dcs of the normal to the plane are proportional to A, B, C, and therefore Condition for parallelism gives.

$\left[\frac{l}{a_1} = \frac{m}{b_1} = \frac{n}{c_1}\right]$ $\qquad \frac{A}{7} = \frac{B}{-7} = \frac{C}{-2} = k$ (say).



$A = 7k,\ B = -7k,\ C = -2k$.

Substituting the value of A, B, C in eqn (i).

$7k(x-3) + -7k(y-2) + -2k(z-1) = 0$.

$k[7(x-3) - 7(y-2) - 2(z-1)] = 0$.

$7x - 21 - 7y + 14 - 2z + 2 = 0$.

$7x - 7y - 2z - 5 = 0$

which is the required equation of plane.

15.

2)

Solution

Total no. of student = 40.

Boy (B) = 40-25 = 15 Girls (G) = 25

$\boxed{\cancel{p = \frac{25}{40} = \frac{5}{8}} \text{ and } \cancel{q = 1-\frac{5}{8} = \frac{3}{8}}.}$

If 2 students are selected at random.

(i) Both are girls.

m = no. of favorable case = $c(25,2) = 300$.

n = no. of possible case = $c(40,2) = 780$

$p(\text{Both are girls}) = \frac{m}{n} = \frac{300}{780} = \frac{5}{13}$.

(ii) First is boy and second is girl.

$= \frac{c(15,1) \times c(25,1)}{c(40,2)}$

$= \frac{15 \times 25}{780} = \frac{25}{52}$

(iii)

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Solution.

→ The steps of solving Gauss elimination method is as:-

(i) Let the system of eqn be.

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$ ------ (i)

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$ ---- (ii)

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$ ---- (iii) where $a_{11} \neq 0$.

Step-1. If $a_{11} = 0$ then we interchang eqn (i) and (ii) or (iii) in order to make $a_{11} \neq 0$.

Step-2. Eliminate x_1 from eqn (i) and (ii) to get an equation with variable x_2 and x_3. Denote this by (iv) and

Step-3. Similarly eliminate x_1 from (i) and (iii). denote it by (v).

Step-4 Eliminate x_2 and x_3 from eqn (iv) and v to get an equation with variable x_3. Denote by (vi).

Now, we have following three type of eqn.

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$ ----- (i)

$a'_{22}x_2 + a'_{23}x_3 = b'_2$ ---- (iv)

$a'_{33}x_3 = b'_3$ ---- (vi).

where a'_{22}, a'_{23}, a'_{33}, b'_2 and b'_3 are Constants. This process is known as forward elimination process.

Step-5. Obtained x_3 from (vi) using value of x_2 in (iv) to get x_2 finally use x_2 and x_3 in eqn (i) to get x_1. This process is called backward substitution.

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Solution

Since the largest absolute value of the Coefficient of x_1 is 4, so we Interchange eqn (i) by eqn (ii).

$$4x_1 + 2x_2 + 3x_3 = 4 \text{ ------ (i)}$$
$$2x_1 + 2x_2 + x_3 = 6 \text{ ------ (ii)}$$
$$x_1 - x_2 + x_3 = 0 \text{ ------ (iii)}$$

multiplying eqn (i) by 1/2 and subtract from (ii).

$$2x_1 + 2x_2 + x_3 = 6.$$
$$- \; 2x_1 + x_2 + 3/2\, x_3 = 2.$$
$$x_2 - \frac{1}{2} x_3 = 4. \text{ ------ (A)}$$

~~Multiplyi eqn (i) by~~ Again multiplying eqn (i) by 1/4. and subtract from. (iii)

$$x_1 - 2x_2 + x_3 = 0.$$
$$- \; x_1 - 1/2\, x_2 + 3/4\, x_3 = 1$$
$$-\frac{3}{2} x_2 + \frac{1}{4} x_3 = -1. \text{ ------ (B)}.$$

Interchanging the two eqn (A) and (B) as the absolute value of Coefficient of x_2 in the 2nd equation is large, we have.

$$-\frac{3}{2} x_2 + \frac{1}{4} x_3 = -1. \text{ ------ (iv)}$$

$$x_2 - \frac{1}{2} x_3 = 4 \text{ ------ (v)}.$$

Again, multiplying eqn (iv) by 2/3 and add with (v).

$$x_2 - 1/2\, x_3 = 4$$
$$-x_2 + 1/6\, x_3 = -2/3.$$
$$-\frac{1}{3} x_3 = \frac{10}{3} \text{ ------ (vi)}.$$

Now, we have the following three equations.

$$4x_1 + 2x_2 + 3x_3 = 4 \text{ ------ } (i)$$

$$-\frac{3}{2}x_2 + \frac{1}{4}x_3 = -1 \text{ ------ } (iv)$$

$$-\frac{1}{3}x_3 = \frac{10}{3} \text{ ------ } (vi)$$

from (vi) $x_3 = -10$

Using the value of x_3 in (iv).

$$-\frac{3}{2}x_2 + \frac{1}{4} \times (-10) = -1$$

$$\therefore x_2 = -1.$$

Using the value of x_2 and x_3 in (i).

$$4x_1 - 2 - 30 = 4$$

$$\therefore x_1 = 9$$

$\therefore$ The Solution is $x_1 = 9$, $x_2 = -1$ and $x_3 = -10$.

17 (a).

Solution

$$f(x) = 1 - (x-1)^{2/3}.$$

for every value of x such that $0 \le x \le 2$, $f(x)$ has a definite value, so $f(x)$ is Continuous in $[0,2]$.

$$f'(x) = \frac{-2}{3(x-1)^{1/3}}$$

which exists for all x such that $0 < x < 2$.

$\therefore f(x)$ is differentiable in open interval $(0,2)$.

Also,

$$f(a) = f(0) = 1 - (0-1)^{2/3} = 1 - 1 = 0$$

$$f(b) = f(2) = 1 - (2-1)^{2/3} = 1 - 1 = 0.$$

$$\therefore f(0) = f(2).$$

So, All condition of Rolle's theorem is satisfied.

Hence, there exist at least a point $c \in (0,2)$ such that

$$f'(c) = 0.$$

$$\frac{-2}{3(c-1)^{1/3}} = 0$$

$$(c-1)^{-1/3} = 0.$$

$$\therefore c = 0 . \in (0,2).$$

Hence, Rolle's theorem is satisfied.

(b).

Solution

Let $y = x^{\sinh^2 x/a}$

Taking log on both side

$$\log y = \log x^{\sinh^2 x/a}.$$

$$\log y = \sinh^2 \frac{x}{a} . \log x.$$

Differentiating both side w.r.t. x

$$\text{or, } \frac{1}{y} \times \frac{dy}{dx} = \sinh^2 \frac{x}{a} \times \frac{d \log x}{dx} + \log x \, \frac{d(\sinh^2 x/a)}{d \sinh x/a} \times \frac{d \sinh x/a}{d\, x/a} \times \frac{d\, x/a}{dx}$$

$$\text{or, } \frac{1}{y} \times \frac{dy}{dx} = \frac{\sinh^2 x/a}{x.} + \log x . 2 \sinh \frac{x}{a} . \cosh \frac{x}{a} \times \frac{1}{a}.$$

$$\text{or, } \frac{1}{y} \times \frac{dy}{dx} = \left[\frac{\sinh^2 \frac{x}{a}}{x} + \frac{\log x}{a} \times \sinh \frac{2x}{a} \right]$$

$$\text{or, } \frac{dy}{dx} = y \left[\frac{1}{x} . \sinh^2 \frac{x}{a} + \frac{1}{a} \log x . \sinh 2x \right]$$

$$\therefore \frac{dy}{dx} = x^{\sinh^2 x/a} \left[\frac{1}{x} \sinh^2 \frac{x}{a} + \frac{1}{a} \log x \sinh 2x \right].$$

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Imp 182 (s.o) $\int \frac{dx}{\sqrt{(x-a)(x-\beta)}}$.

put $y^2 = x-\beta$.

$2y\,dy = dx$.

Now

$$I = 2\int \frac{y\,dy}{\sqrt{(y^2+\beta-a)y^2}}$$

$$= 2\int \frac{dy}{\sqrt{y^2+(\sqrt{\beta-a})^2}}$$

$$= 2\log(y+\sqrt{y^2+\beta-a})$$

$$= 2\log(\sqrt{x-\beta}+\sqrt{(\sqrt{x-\beta})^2+\beta-a})$$

$$= 2\log(\sqrt{x-\beta}+\sqrt{x-a}).$$

8 (b) $\int \frac{dx}{2\sin x+3\cos x}$.

put $2 = r\cos\theta$ and $3 = r\sin\theta$.

$r^2 = 2^2+3^2 = 13$.

$\therefore r = \sqrt{13}$.

$\tan\theta = \frac{y}{x} = \frac{3}{2}$. $\therefore \theta = \tan^{-1}\frac{3}{2}$

$$I = \frac{1}{r}\int \frac{dx}{\cos\theta\sin x+\sin\theta\cos x}$$

$$= \frac{1}{r}\int \frac{dx}{\sin(\theta+x)}$$

$$= \frac{1}{r}\int \operatorname{cosec}(x+\theta)\,dx$$

$$= \frac{1}{r}\log\left(\tan\frac{(x+\theta)}{2}\right)+c.$$

$$= \frac{1}{\sqrt{13}}\log\left(\tan\frac{1}{2}\left(x+\tan^{-1}\frac{3}{2}\right)\right)+c.$$